

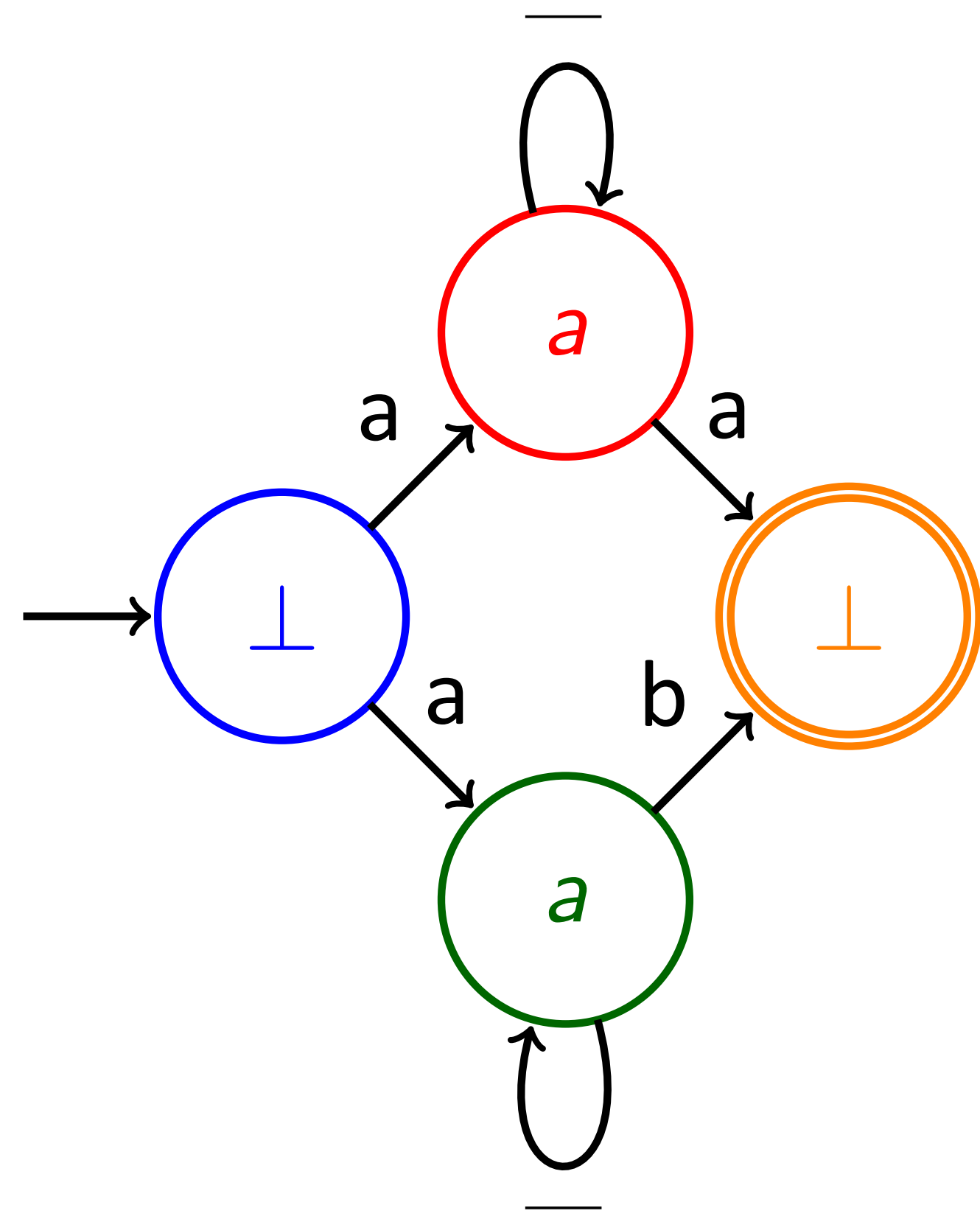
Multidimensional linear recursive sequences and universality of unambiguous register automata

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Unambiguous register automata w/o guessing



- $a, b \in \mathbb{N}$, compared with $=$ and \neq .
- Unambiguity: ≤ 1 accepting run per word.
- Non-guessing: registers hold previous inputs.

Universality problem: $\mathcal{L}(\mathcal{A}) = \mathbb{N}^*$?

- Undecidable in general.
- In EXPSPACE for fixed #registers for unambiguous automata without guessing [3].
- New: Generic reduction from inclusion/equivalence to universality (!).

Our approach

Key idea (inspired from [1, 5])

Count the number of orbits $f(n, k)$ of initial runs of length n and k distinct letters.

- Multidimensional linear recurrence sequence with *univariate* polynomial coefficients (1).
- Universality \rightarrow zeroness of $S(n, k) - f(n, k)$.
- Elimination using non-commutative Ore polynomials [4]; yields (2) \rightarrow 4-EXPSPACE complexity.

Improving the complexity

Second key idea

Use linear non-commutative algebra.

- Hermite forms of Ore polynomial matrices [2].
- Careful complexity analysis yields EXPSPACE.

Future work

- Extend to total order, timed automata, grammars, etc... (decidability still unknown).

Example

$$\begin{cases} f_{\bullet}(n+1, k+1) = 0 \\ f_{\bullet}(n+1, k+1) = f_{\bullet}(n, k+1) + f_{\bullet}(n, k) + k \cdot f_{\bullet}(n, k+1) \\ f_{\bullet}(n+1, k+1) = f_{\bullet}(n, k+1) + f_{\bullet}(n, k) + k \cdot f_{\bullet}(n, k+1) \\ f_{\bullet}(n+1, k+1) = f_{\bullet}(n, k+1) + f_{\bullet}(n, k) + (k-1) \cdot f_{\bullet}(n, k+1) \end{cases} \quad (1)$$

$$(k^2 - 5k + 6) \cdot f_{\bullet}(n+3, k+3) + (2k+2) \cdot f_{\bullet}(n+3, k+2) - (3k+3) \cdot f_{\bullet} = 0 \quad (2)$$

References

- [1] N. Chomsky and M. P. Schützenberger. “The Algebraic Theory of Context-Free Languages”. 1963.
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- [4] Oystein Ore. “Theory of Non-Commutative Polynomials”. 1933.
- [5] Richard E. Stearns and Harry B. Hunt. “On the equivalence and containment problems for unambiguous regular expressions, grammars, and automata”. 1981.