Universality of unambiguous register automata

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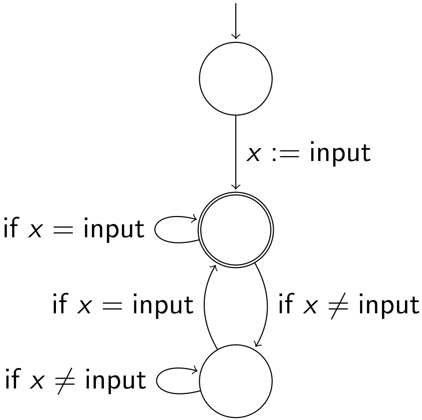
Non-guessing register automata

x is a register that stores values from $\mathbb{N}$



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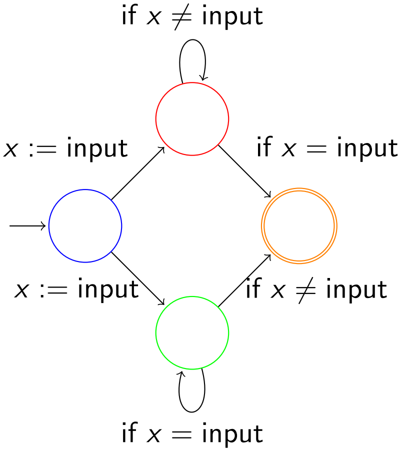

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$$\text{Inclusion} \iff \text{Equivalence} \iff \text{Universality}$$

This is quite surprising: for DCFG, Universality is decidable but not Inclusion!

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(Undecidable in general)

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$$F_s(n,k) = \text{number of orbits of runs that end in } s \\ \text{with length } n \text{ and with } k \text{ different letters}$$

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$$\begin{cases} f_{\color{blue}\bullet}(n+1,k+1) = & 0 \\ f_{\color{red}\bullet}(n+1,k+1) = & f_{\color{blue}\bullet}(n,k) + (k+1)\cdot f_{\color{blue}\bullet}(n,k+1) + f_{\color{red}\bullet}(n,k) + k\cdot f_{\color{red}\bullet}(n,k+1) \\ f_{\color{green}\bullet}(n+1,k+1) = & f_{\color{blue}\bullet}(n,k) + (k+1)\cdot f_{\color{blue}\bullet}(n,k+1) + f_{\color{green}\bullet}(n,k+1) \\ f_{\color{orange}\bullet}(n+1,k+1) = & f_{\color{red}\bullet}(n,k+1) + f_{\color{green}\bullet}(n,k) + k\cdot f_{\color{green}\bullet}(n,k+1) \\ S(n+1,k+1) = & S(n,k) + (k+1)\cdot S(n,k+1) \\ g(n+1,k+1) = & S(n,k) - f_{\color{orange}\bullet}(n,k) \end{cases}$$

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$$\left\{\begin{array}{rrrrrl}
(\partial_1\partial_2)\cdot f_{\color{blue}\bullet} & & & & & =0\\
-\partial_2\cdot f_{\color{blue}\bullet} & +(\partial_1\partial_2-(k+1)\partial_2-1)\cdot f_{\color{red}\bullet} & & & & =0\\
-\partial_2\cdot f_{\color{blue}\bullet} & & + & (\partial_1\partial_2-\partial_2)\cdot f_{\color{green}\bullet} & & =0\\
 & -\partial_2\cdot f_{\color{red}\bullet} & + & (-(k-1)\partial_2-1)\cdot f_{\color{green}\bullet} & +(\partial_1\partial_2)\cdot f_{\color{orange}\bullet} & =0\\
 & (\partial_1\partial_2-(k+1)\partial_2-1)\cdot S & & & & =0\\
 & \partial_1\partial_2 g-S+f_{\color{orange}\bullet} & & & & =0
\end{array}\right.$$

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$$((k^2 - 5k + 6)\partial_1^3\partial_2^3 + (2k + 2)\partial_1^3\partial_2^2 - 2\partial_1^2\partial_2 + \partial_2^2 - (3k + 3)) \cdot g = 0$$

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Yields a 4-EXP-SPACE algorithm.

Improving the complexity

Idea: Instead of removing one variable at a time, invert the matrix using (non-commutative) linear algebra.

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- Put matrices of operators in a triangular form (Hermite form):

$$\begin{pmatrix} (\partial_1 - 1)\partial_2 & -\partial_2 \\ -k\partial_2 - 1 & \partial_1\partial_2 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & (\frac{k}{\partial_1 - 1} - \partial_1)\partial_2 \\ 0 & \partial_2^2 - \frac{1}{\partial_1^2 - \partial_1 - (k+1)}\partial_2 \end{pmatrix}$$

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- Exponential bound on the coefficient of the Hermite form [Giesbrecht,Kim 11].

Main theorem

Theorem [B.,Clemente 20]

The universality problem for unambiguous register automata is decidable in 2EXP-TIME.

Conclusion

- Improving the complexity. Monicity conjecture: monic cancelling relations suffice:

$$(\cancel{(k^2-5k+6)}\partial_1^3\partial_2^3 + (2k+2)\partial_1^3\partial_2^2 - 2\partial_1^2\partial_2 + \partial_2^2 - (3k+3)) \cdot g = 0$$

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- Extend to other structures: other atoms, timed automata, pushdown automata. . .
- Extend to weighted automata.

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